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NATURAL CONVECTION FROM A VERTICAL FLAT PLATE IN THE LOW GRASHOF NUMBER RANGE

J. GRYZAGORIDIS
Department of Mechanical Engineering, University of Cape Town, Cape Town, South Africa

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NOMENCLATURE

A, surface area of the vertical plate [m^2];
C_p, specific heat of the fluid [J/kg deg];
g, acceleration due to gravity [m/s^2];
Gr, Grashof number, $\rho^2 \dfrac{\beta g(T_w - T_\infty)L^3}{\mu^2}$;
h_c, free convective coefficient [W/m^2 deg];
h_r, radiation coefficient [W/m^2 deg];
k, thermal conductivity of the fluid [W/m deg];
L, length of the plate [m];
Nu, Nusselt number, $Nu = \dfrac{h_c L}{k}$;
Pr, Prandtl number, $Pr = \dfrac{C_p \mu}{k}$;
Q, heat transfer rate [W];
T, temperature [°K];
U, combined radiation and convection heat transfer coefficient [W/m^2 deg].

Greek symbols
β, coefficient of thermal expansion of fluid [deg^{-1}];
δ, boundary layer thickness [m];
ε, emmissivity of the plate's surface [dimensionless];
μ, absolute viscosity of the fluid [kg/ms];
ρ, density of the fluid [kg/m^3];
σ, Stefan–Boltzman constant [W/m^2 deg^4].

Subscripts
c, chamber conditions;
W, wall conditions;
∞, ambient conditions.

INTRODUCTION

FOR AVERAGE heat transfer rates from a vertical heated plate in laminar free convection the empirical relationship

$$Nu = 0{\cdot}555(Gr.Pr)^{\frac{1}{4}} \tag{1}$$

is recommended for use, by Schlichting [1] and Kreith [2] for the Grashof number range of 10^5–10^8. Equation (1) compares very favourably with the classical boundary layer solution by Schmidt and Beckman [3] for local heat transfer rates,

$$Nu = 0{\cdot}387\,(Gr.Pr)^{\frac{1}{4}} \tag{2}$$

if we assume that we can obtain average rates by integrating equation (2) between the limits of $x = 0$ and $x = L$ and dividing by L yielding

$$Nu = 0{\cdot}516\,(Gr.Pr)^{\frac{1}{4}}. \tag{3}$$

In the moderate and low Grashof number range, i.e. $Gr < 10^5$, both equations (1) and (2) are widely disputed.

Saunders [4] has shown excellent agreement with equation (1) as far down as $Gr \times 10^5$, however below this range there is a marked departure (see Fig. 1). His explanation, for the lack of correlation below $Gr = 10^5$, that the convective currents become insignificant and the energy is predominently transferred by pure conduction, is quite valid. There is however some doubt whether the influence of pure conduction is felt at such high numbers as shown in his results. Suriano and Yang [5] stated that the boundary layer effects on free convection become predominant only in the region above $Gr \approx 10^3$ and presented a limiting Nusselt number of 1·078.

Experimental data for local heat-transfer rates reported by Cheesewright [6] and Goldstein and Eckert [7] suggest aggreement with the boundary layer solution down to $Gr \simeq 10$ and that the limiting Nusselt number may be less than that reported by Suriano and Yang [5].

For overall heat transfer rates, bearing in mind Saunders' [4] results, it is suggested that detailed experimental data is needed to ascertain the validity of the boundary layer concepts, in the moderate and low Grashof number range. Brodowicz [8] has shown that the lack of agreement observed between the boundary layer solution and experimental data is due to conditions under which experiments are run. When measurements for overall heat-transfer rates are being made, the influence of the leading edge of the plate may be significant.

EXPERIMENTAL TECHNIQUE

In order to obtain a wide range of Grashof numbers it was decided to use a vacuum chamber where the density of the air would be reduced, thus lowering the Grashof number. In investigating the low Grashof number range two plates were used having the dimensions of 3 × 2-in. and 2 × 3-in. respectively. Also to ascertain that there would be no effect of the size of the plates to the enclosure, three different vacuum chambers were employed as shown in Fig. 2. The plates were suspended vertically in the chambers and consisted of two copper plates bolted together with a heating element between them. The apparatus will be described in detail in [9].

The method adopted for the measurement of average heat-transfer rates was as follows:

heat input equivalent to electric power	=	heat lost by convection from the plate	+	heat lost by radiation from the plate.

There were negligible conduction losses since the plate was suspended by thin wires inside the vacuum chamber.

From the defining equation

$$Q = UA(T_w - T_\infty) \tag{4}$$

the overall heat-transfer coefficient U, was determined.

The radiation coefficient was determined from

$$h_r = \frac{\sigma \varepsilon A(T_w^4 - T_c^4)}{(T_w - T_\infty)}. \tag{5}$$

Here the assumption is that the plate, a "grey body", radiates energy which is absorbed by the vacuum chamber walls, a "black body", simulated by painting the inside of the chamber with matt black paint.

The free convective coefficient was calculated from

$$h_c = U - h_r. \tag{6}$$

The properties in the dimensionless groups (Nu, Gr, Pr) were determined from the temperature and pressure measurements in the vacuum chamber and the dimensionless groups were evaluated at the representative temperature of the level inside the boundary layer, recommended by Sparrow and Gregg [10].

$$T_r = T_w - 0{\cdot}38\,(T_w - T_\infty). \tag{7}$$

Due to the construction of the plates neither conditions of uniform heat flux or uniform temperature were obtained, but rather an in-between state, however in the final evaluation of the results the vertical plate was assumed to be of uniform temperature, indicated by a thermocouple placed half way along its height. Preliminary tests on the plates showed that the average temperature was very near the one indicated by the thermocouple placed at the mid point of the vertical plate. This is in accordance with Sparrow and Gregg [11] who showed that when an average Nusselt number is calculated for a plate with uniform heat flux using the temperature difference halfway along the height, the results are remarkably close to those obtained when using a uniform temperature plate.

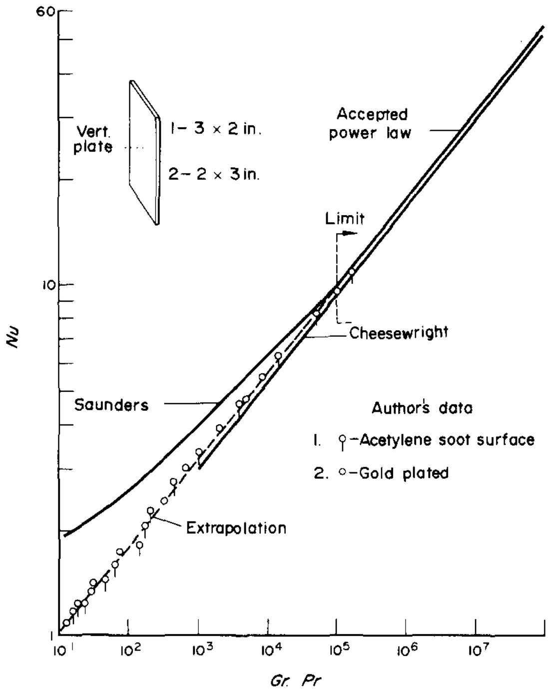

FIG. 1. Experimental results of average heat-transfer rates by free convection from a vertical plate.

The emissivity of the copper plates greatly influences the radiation coefficient and consequently the free convection coefficient from the plate [see equations (5) and (6)]. Care was therefore taken to use independently measured emissivity values of different surfaces for the experimental plates.

It is an accepted fact that for laminar boundary layer in natural convection

$$\delta/L \propto Gr^{-\frac{1}{4}} \tag{8}$$

which indicates that $\delta \to \infty$ if $Gr \to 0$ for a constant L. Thus care was taken to ensure that the size of the enclosure was adequate for the plates' height so there would be no interference of the boundary layer with the sides of the vacuum chamber. This was verfied by temperature probing the field between the plate's surface and the chamber's walls. This technique also served in verifying the bulk liquid temperature measurements ensuring its uniformity well away from the plate.

RESULTS AND DISCUSSION

Figure 1 shows in dimensionless form the results for average heat-transfer measurements in the laminar region.

For comparison purposes the accepted power law relationship [equation (1)] is extrapolated below its recommended limit and the curve by Saunders [4] is included. The author has taken the liberty to convert Cheesewright's [6] data to average values and include the results in Fig. 1. It can be seen that at no point has departure from the power law relationship been observed as shown by Saunders [4].

Since the data reported by Saunders [4] is insufficient to perform an analysis one may suggest that his departure from the power-law relationship is due to his failure to recognize that the boundary layer thickness increases rapidly at low Grashof numbers. Thus if his temperature measuring probe was within the boundary layer it would indicate smaller temperature differences between the plate and the bulk air and hence higher heat rates.

In Fig. 1 the lack of correlation with the "pure conduction limit" reported by Suriano and Yang [5] is also observed. It is unfortunate that lower Grashof numbers than $Gr = 10$ could not be obtained, so the most that can be said is that the pure condition limit occurs at Grashof numbers well below 10 and that the limiting Nusselt number is less than 1·078.

The results also show that the leading edge of the plate does not influence greatly the average heat-transfer rates from the vertical plate. This is in accordance with Scherberg [12] who has shown that for the case of a constant temperature plate the leading edge only affects the relative vertical position of the boundary layer and not its thermal or velocity form. From the interference photographs shown by Brodowicz [8] it is also seen that the flow ahead of the leading edge depends on the physical thickness of the plate under consideration and will only become important if the plate's height is reduced to such an extent that the portion of the boundary layer ahead of the plate is of noticeable percentage of the overall boundary layer exhibited around it. It is also worth mentioning along the same lines, that Saunder's [4] results in the lower Grashof range were obtained using plates having extremely small heights (2·5 and 0·325 cm).

In view of the results obtained in this investigation, it is suggested that the power-law relationship can be used with sufficient accuracy in the laminar region and its units be increased from $10^5 < Gr.Pr < 10^8$ to $10 < Gr.Pr < 10^8$.

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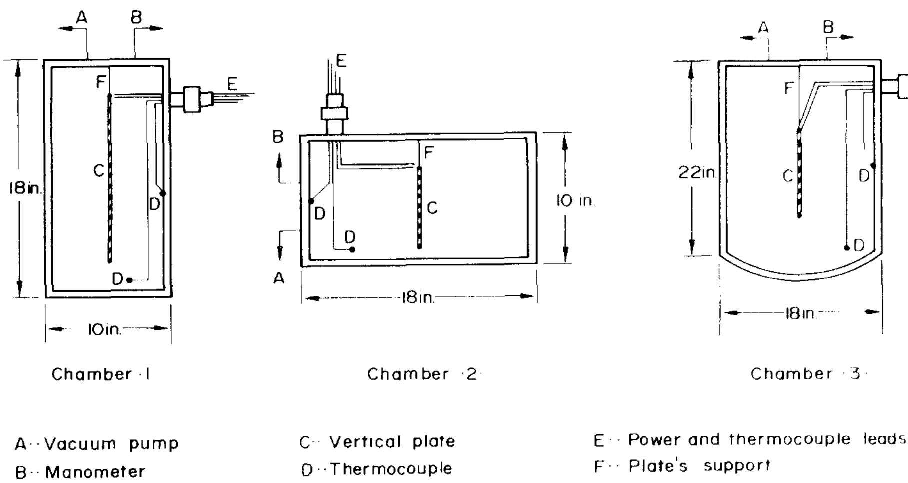

FIG. 2. Vacuum chambers used in the investigation showing position of the vertical plates and main temperature measuring probes.

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ON THE BREAK-UP OF THIN LIQUID LAYERS FLOWING ALONG A SURFACE

E. RUCKENSTEIN

Clarkson College of Technology, Potsdam, New York 13676, U.S.A.

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NOMENCLATURE

c, concentration of the more volatile component;
Δc, concentration difference between that at the free interface far from the leading edge and at the leading edge;
g, acceleration of gravity;
k, thermal conductivity;
q, heat flux;
T_b, boiling temperature;
ΔT, temperature difference between that at the free interface far from the leading edge of the rivulet and the wall;
u, velocity component in the direction of the main flow;
x, mole fraction of the more volatile component;
y, distance from the wall;
α, proportionality constant, α_0—value of α for $\theta = 0$;
Γ, volumetric flow rate per unit perimeter;
Γ_c, critical value of Γ;
δ, film thickness;
δ_0, film thickness far from the leading edge of a patch or rivulet;
η, viscosity of the liquid;
ν, kinematic viscosity of the liquid;
ρ, liquid density;
σ, liquid–gas surface tension;
σ_{sg}, solid–gas surface tension;
σ_{ls}, liquid–solid surface tension;
τ_0, shear stress;
θ, dynamic angle (see Figs. 2 and 3);
θ_e, static angle (See Fig. 1).

INTRODUCTION

THE PROBLEM of break-up of thin liquid films flowing along a surface has been examined in the literature from two different points of view. Some of the authors have given attention to the occurrence of a dry patch [1–3] and have established equations for the minimum flow-rate at which the surface is completely wetted. In this case the surface tension has an important part. Other authors have explained the occurrence of rivulets in conditions of heat or mass transfer by means of the Marangoni effect (surface tension gradient) [4–7] but have treated the problem in a qualitative manner. There exists a single paper [8] in which the Marangoni effect is taken into account in a quantitative manner but only for correcting the results obtained in connection with the stability of a dry patch. In the present paper both the stability of patches and that of rivulets will be examined.

Experiment shows that, when a thin liquid film is flowing in isothermal conditions along a vertical wall, dry patches can form if the flow-rate is sufficiently small. A significant contribution concerning the analysis of stability of a dry patch is due to Hartley and Murgatroyd [1] and Murgatroyd [2]. The stability condition of a patch is obtained by equating the surface forces and upstream kinetic energy of the liquid. Using for the surface forces the expression $\sigma(1 - \cos\theta)$